SELECTIVITY OF AN ACOUSTIC DETECTOR IN A TURBULENT STREAM

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Relationships obtained for piezoelectric systems are extended to an arbitrary kind of electroacoustic transducers. Expressions are obtained for the pulsation of transducers with an arbitrary distribution of oscillations over the detector surface.

The determination of the statistical characteristics of turbulent pressure fields on the surface of elastic structures (the space–time correlation function $R(\vec{\varepsilon}, \tau)$ and the mutual frequency spectrum over the space of the turbulent pressure pulsations $\Gamma(\vec{\varepsilon}, \omega)$) is performed by miniature pressure detectors, electroacoustic transducers, mounted at minimal spatial separations flush with the surface around which the turbulent stream flows.

Evidently, only an "idealized" point detector will record the pressure pulsation without distortion. Acoustic microdetectors, known at this time, cause a systematic error in measuring turbulent pressure pulsations, whose interval of coherence is considerably less than the geometric size of the transducers, because of the mutual cancellation of the fine-scale pulsations at the sensitive detector surface. To a certain degree, an analogous situation occurs in the measurement of the acoustic pressure when the detector size and the acoustic wavelength in the medium agree: the error because of interference between the incident and diffracted waves on the detector surface should be taken into account in a broad frequency range.

Experimental investigations of the statistical structure of a turbulent stream have attracted the attention of researchers to questions of the acoustic metrology of turbulence [1-9]. We have introduced this special designation to combine the diverse effects of the geometric dimensions and the shape and orientation of the acoustic detector on its sensitivity to turbulent pressure pulsations. The change in sensitivity of an acoustic detector in a turbulent stream [2, 6] is determined by the process of shaping the electrical output of the transducer under conditions of incoherent detection of the turbulent pulsations, i.e., the energetic summation of the electrical signals originating in the detector under the effect of "vortices" being propagated along it.

The electromechanical conversion of turbulent pressure pulsations by a piezoelectric detector has been investigated earlier [1]. For the acoustic metrology of turbulence it is interesting to extend the results obtained to an arbitrary mechanism of electromechanical conversion and to investigate the influence of structural factors on the operation of the detector in a stream.

Let us write down the initial relationship permitting formulation of the problem of distortion of the spectral density of turbulent pulsations in terms of the sensitivity of an acoustic detector [1]:

$$\frac{P_{\text{obs}}(\omega)}{P_{\text{tr}}(\omega)} = \frac{\gamma_{\tau}^{2}}{\gamma_{\text{sf}}^{2}}. \tag{1}$$

The "field" sensitivity of a detector γ_{sf} can be found experimentally in the field of a plane traveling sound wave on the axis of the directivity characteristic. The detector sensitivity in the stream γ_{T} is determined by means the formula [6]

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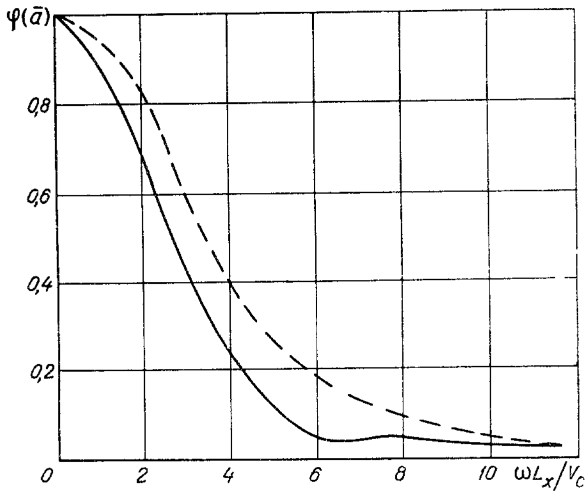

Fig. 1

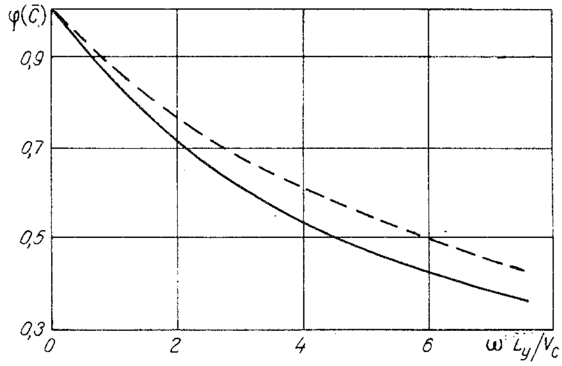

Fig. 2

Fig. 1. Frequency dependence of the coefficient of longitudinal selectivity [dashed line) flexible detector; solid line) piston detector].

Fig. 2. Frequency dependence of the coefficient of transverse selectivity (notation of the curves is the same as in Fig. 1).

$$\gamma_T^2 = \frac{U_T(\omega)}{P_{tr}(\omega)}. \tag{2}$$

Transforming (1)

$$\frac{\gamma_T^2}{\gamma_{sf}^2} = \frac{U_T(\omega)}{U_{sf}(\omega)}, \tag{3}$$

we find that in order to determine the pulsation sensitivity of an acoustic detector the field of electrical signals U ($\mathbf{x}$, ω) at an arbitrary point of the detector surface in the stream should be examined, and then passage should be made in the customary manner to the spectral density of the electrical output $U_T(\omega)$.

The electrical signal developed by the transducer in a plane sound wave field is known

$$U_{sf}(\omega) = \left(\frac{z_{el}}{z_{mech}} NF_{eq}\right)^2. \tag{4}$$

The relationship (4) is valid in the frequency domain far from transducer resonance. An energy source for the transducer is the pressure field with the motive force F_{eq}:

$$F_{eq} = P(\omega) S_{surf} \tag{5}$$

$$S_{surf} = \int_S f(\mathbf{x})\, d\mathbf{x}. \tag{6}$$

The function f(x) defines the given sensitivity distribution over the detector surface S.

When recording turbulent noise only the generalized perturbation F_{eq}, which becomes a random function of the coordinates and time, changes in (4), and we utilize the connection between the input (F_{eq}) and output (vibration) spectra of a linear system to determine it. Drawing upon the expression for plate vibrations in a stream [1], we obtain for the generalized perturbation after transformation

$$F_{eq}(\mathbf{x}, \omega) = f(\mathbf{x}) \int_S P_T(\mathbf{x}_0, \omega) f(\mathbf{x}_0)\, d\mathbf{x}_0. \tag{7}$$

Substituting (7) into (4) affords the possibility of finding the field of the electrical signal at an arbitrary point x of the detector surface

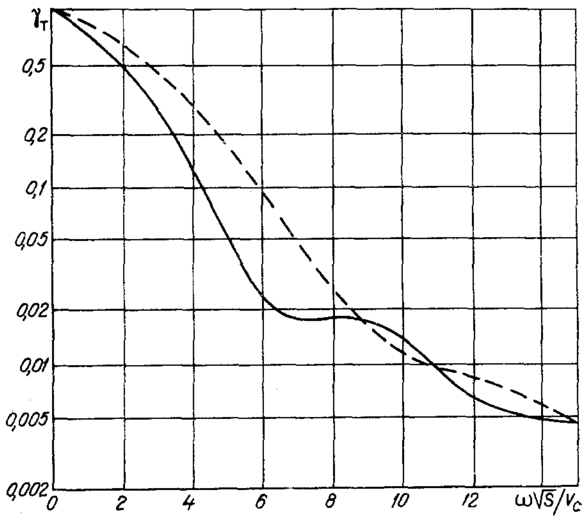

Fig. 3

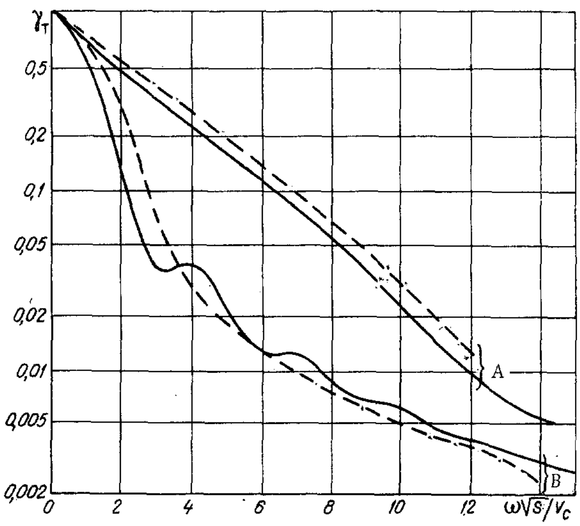

Fig. 4

Fig. 3. Frequency dependence of the pulsation sensitivity of a square detector (notation of the curves the same as in Fig. 1).

Fig. 4. Frequency dependence of the pulsation sensitivity of a rectangular detector with 5 : 1 side ratio and the detector oriented across (A) and along (B) the stream (notation of the curves the same as in Fig. 1).

$$U_T(\mathbf{x}, \omega) = \frac{z_{\text{el}}}{z_{\text{mech}}} N f(\mathbf{x}) \int_S P_T(\mathbf{x}_0, \omega) f(\mathbf{x}_0)\, d\mathbf{x}_0 \tag{8}$$

when it is excited at a point $\mathbf{x}_0$ by the turbulent pressure pulsations $P_T(\mathbf{x}_0, \omega)$.

Shaping of the electrical output of an acoustic detector in a turbulent stream recalls the process of spatial filtration in the formation of an image in optics [10], and is defined by the spatial coherence of the electrical signals (8). Evidently, we should turn to the mutual correlation of the turbulent pressure pulsations in order to find the total electrical output of the detector.

The mutual frequency spectrum over space $\Gamma(\mathbf{x}_0 - \mathbf{x}_0', \omega)$ is determined experimentally in aero- and hydrodynamic wind tunnels [11]:

$$\begin{gathered}\Gamma(\mathbf{x}_0 - \mathbf{x}_0', \omega) = P(\omega) \exp(-a\,|x_0 - x_0'| - c\,|y_0 - y_0'|) \cos b\,(x_0 - x_0'),\\ a = 0.087b; \quad c = 0.557b; \quad b = \omega/V_c.\end{gathered} \tag{9}$$

Let us form the mutual spectrum $\Gamma_U(\mathbf{x}, \mathbf{x}_1, \omega)$ of the electrical signals at the points $\mathbf{x}$ and $\mathbf{x}_1$ under the effect of pulsations at the points $\mathbf{x}_0$ and $\mathbf{x}_0'$; then denoting the time average by a bar, we obtain:

$$\begin{aligned}\Gamma_U(\mathbf{x}, \mathbf{x}_1, \omega) &= \overline{U_T(\mathbf{x}, \omega)\, U_T^*(\mathbf{x}_1, \omega)}\\ &= \left(\frac{z_{\text{el}}}{z_{\text{mech}}} N\right)^2 f(\mathbf{x}) f(\mathbf{x}_1) \iint_{SS'} \Gamma(\mathbf{x}_0 - \mathbf{x}_0', \omega) f(\mathbf{x}_0) f(\mathbf{x}_0')\, d\mathbf{x}_0 d\mathbf{x}_0'.\end{aligned} \tag{10}$$

Averaging the expression obtained over the detecting surface, summing over all points of observation, $\mathbf{x}$ and $\mathbf{x}_1$, we find the spectral density of the total electrical output of the acoustic detector in the stream

$$U_T(\omega) = \iint_{SS'} \Gamma_U(\mathbf{x}, \mathbf{x}_1, \omega)\, d\mathbf{x} d\mathbf{x}_1. \tag{11}$$

After substituting (9) and (10) into (11), and taking account of the orthonormalization condition of the oscillations $f(\mathbf{x})$, we obtain

$$U_T(\omega) = \left(\frac{z_{\text{el}}}{z_{\text{mech}}} N\right)^2 P(\omega) S^2 \varphi(\bar{a})\, \varphi(\bar{c}). \tag{12}$$

Equating (12) and (4), taking account of (5) and (6), we find that the dimensionless functions $\varphi(\bar{a})$ and $\varphi(\bar{c})$ in (12) define the selectivity of the detector in the stream. The coefficients of longitudinal $\varphi(\bar{a})$ and transverse $\varphi(c)$ selectivity for a rectangular detector with sides L_x and L_y are:

$$\varphi(\bar{a}) = \frac{1}{L_x^2}\int_0^{L_x}\int_0^{L_x} \exp(-a\,|x_0 - x_0'|)\cos b\,(x_0 - x_0')\,f(x_0)\,f(x_0')\,dx_0 dx_0',$$

$$\varphi(\bar{c}) = \frac{1}{L_y^2}\int_0^{L_y}\int_0^{L_y} \exp(-c\,|y_0 - y_0'|)\,f(y_0)\,f(y_0')\,dy_0 dy_0',$$

$$\bar{a} = aL_x; \qquad \bar{c} = cL_y. \tag{13}$$

The distribution function of the oscillations f(x) takes account of the influence of the structural factors; for real detectors it should be determined experimentally, by a miniature accelerometer, for example.

Let us simplify the expressions for the selectivity coefficients (13) by passing to the variable $\vec{\varepsilon} = |x_0 - x_0'|$. After evaluation of the inner integrals with respect to the variable x_0, we obtain:

$$\varphi(\bar{a}) = \int_0^{L_x} \exp(-a\varepsilon_x)\cos b\varepsilon_x \Theta(\varepsilon_x)\,d\varepsilon_x,$$

$$\varphi(\bar{c}) = \int_0^{L_y} \exp(-c\varepsilon_y)\,\Theta(\varepsilon_y)\,d\varepsilon_y. \tag{14}$$

The functions $\Theta(\vec{\varepsilon})$ are called the spatial convolution of the shapes of the oscillations, which are for hinged conditions on the contour

$$\Theta(\vec{\varepsilon}) = \frac{L-\varepsilon}{L^2}\cos\frac{\pi}{L}\varepsilon + \frac{1}{\pi L}\sin\frac{\pi}{L}\varepsilon. \tag{15}$$

After substitution of (15) into (14) and simple calculations, we obtain for the coefficient of transverse selectivity

$$\varphi(\bar{c}) = 2\left\{\frac{\bar{c}+1+e^{-\bar{c}}}{\bar{c}^2+\pi^2} + \frac{(\pi^2-\bar{c}^2)(e^{-\bar{c}}+1)}{(\bar{c}^2+\pi^2)^2}\right\}. \tag{16}$$

The expression for the coefficient of longitudinal selectivity is awkward, and is not presented here. In the particular case of a piston-type detector, there is no distribution of oscillations (f(x) = 1) and (13)-(14) go over into the Corcos formula [3].

Shown in Figs. 1 and 2 are the frequency dependences of the coefficients of longitudinal and transverse selectivity. It is interesting to note that a piston-type detector manifests greater selectivity in a stream. It is seen that the selectivity drops as the detector size or the frequency of analysis grows (cancellation of the fine-scale pulsations).

After substituting (4) and (12) into (3), we find the pulsation sensitivity of an acoustic detector

$$\gamma_T^2 = \gamma_{e1}^2\alpha^2\varphi(\bar{a})\,\varphi(\bar{c}). \tag{17}$$

The coefficient α equals the ratio between the real and equivalent surfaces $\alpha = S/S_{surf}$.

Frequency dependences of the pulsation sensitivity are shown in Figs. 3 and 4 for detectors with an m = 1 and m = 5 side ratio. The passage from the dimensionless frequencies $\omega L/V_c$ to $\omega\sqrt{S/V_c}$ is accomplished by means of the formulas:

$$m = L_x/L_y;$$

$$\bar{\omega} = \frac{\omega L_x}{V_c} = \frac{\omega\sqrt{S}}{V_c}m^{1/2};$$

$$\bar{\omega} = \frac{\omega L_y}{V_c} = \frac{\omega\sqrt{S}}{V_c}m^{-1/2}. \tag{18}$$

It is seen that the detector orientation and shape control its selectivity in the stream. The pulsation sensitivity drops more rapidly when the detector is oriented with its largest dimension along the stream. The effect is magnified if the shape is changed for the same detector surface area by increasing the ratio m between the longitudinal and transverse sides. The pulsation sensitivity of detectors with a distribution of oscillations is higher than in piston type. The experiment was conducted under deep sea conditions on a floating unit analogous to that described in [2, 12] with detectors of different shape, area, and orientation in the stream.

The results of the experiment [13] agree with the theory developed above.

NOTATION

P	is the pressure, N/m^2;
ω	is the circular frequency, Hz;
ε	is the spatial separation between two points in the stream, m;
τ	is the time, sec;
γ	is the detector sensitivity, $V/N/m^2$;
U	is the electrical voltage, V;
V_c	is the velocity of turbulent pulsation transport, m/sec;
$f(x)$	is the shape of the oscillations, m^{-1};
$\Theta(\varepsilon)$	is the spatial convolution of the oscillation shapes, m^{-1};
b	is the turbulence wave number, m^{-1};
N	is the coefficient of electromechanical transformation of the transducer.

Subscripts

T	denotes the value of the parameter in the turbulent stream;
sf	denotes the sound field;
obs	denotes the observed value of the parameter;
tr	denotes the true value of the parameter.

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